Chapter 3
Risk and Return: Part II
ANSWERS TO BEGINNING-OF-CHAPTER QUESTIONS

We do not normally cover the material in Chapter 3 in depth in our intermediate financial management course—this treatment is reserved for the investments course—but it is useful for students to recognize that the CAPM results were derived under some restrictive assumptions and hence the derived equations do not necessarily describe how returns are established in the real world.

3-1 In finance theory, the value of an investment is found as the PV of the asset’s expected stream of cash flows. The CAPM is a theory that specifies how the discount rate in the valuation equation should be determined. Although the theory is quite complex and has many component parts, its “bottom line” is the SML equation, often called the CAPM equation:

\[ k_s = k_f + b(k_m - k_f) \]

The CAPM is the product of a number of different researchers, but its principal developer was William Sharpe. Sharpe was a UCLA doctoral student employed by Rand Corporation in the early 1960s while he worked on his dissertation. Harry Markowitz, who in the early 1950s had developed the concept of efficient portfolios as illustrated in Figure 3-3 and Equation 3-4, also worked at Rand, and he helped Sharpe formulate his ideas. Sharpe expanded Markowitz’s portfolio theory to include the riskless rate of return and thus the SML as shown in Figure 3-7. Sharpe also recognized that the “relevant risk” of any individual stock could be measured by its beta coefficient, and thus he developed the SML as shown in Figure 3-10 and Equation 3-8.

A number of simplifying assumptions, including the following, were made in order to derive the CAPM:

1. Investors focus on a single holding period.
2. Investors can borrow or lend unlimited amounts at the riskless rate.
3. Investors have homogeneous expectations regarding the returns of different assets.
4. There are no transactions costs.
5. There are no taxes.
6. The buy/sell decisions of any single investor do not influence market prices.

Since these assumptions are not true in the real world, the SML equation may not be correct, i.e., investors may not establish discount rates in accordance with the CAPM.

Answers and Solutions: 3-1
Sharpe and Markowitz received the Nobel Prize for their work, which has had a profound effect on the finance profession.

3-2 Covariance shows how two variables move in relation to one another. There are different "states of nature," each with a probability of occurrence, and a return on each asset under each state. This probability data can be used to determine the SD of returns for each asset, the variance of those returns, and the correlation between returns on the different assets.

Equation 3-2 in the text can be used to calculate the covariance. Markowitz noted that the returns on a portfolio are simply a weighted average of the expected returns to the individual stocks in the portfolio, but the riskiness of the portfolio as measured by the portfolio's SD is a function of the covariance of returns between the securities. See Equation 3-1 for the SD of a 2-asset portfolio, and note that terms can be added to that equation to deal with the n-asset case.

The intuition behind the effects of covariance are straightforward:
1) If an asset is positively correlated with other assets, the higher the SD of its returns, the more risk it adds to the portfolio.
2) The higher the correlation coefficient between asset returns, the greater the risk of a portfolio of those assets. 
3) If some asset happens to be negatively correlated with other assets, then including it in the portfolio will reduce the risk of the portfolio, and the greater the negatively correlated asset's SD, the greater its risk-reducing effects.
4) If a stock does not contribute much risk to a portfolio, then the stock will not be especially risky to a well-diversified investor.

Sharpe and Markowitz formalized all this so that mathematical models could be developed and programmed into computers to actually determine the inputs and establish efficient portfolios.

3-3 An efficient portfolio is one that produces the highest expected return for any given level of risk. Markowitz showed how to find the frontier of risk and returns for stocks; see Figure 3-3. Only portfolios on the frontier are efficient. Sharpe added the riskless asset return and noted that returns on a line connecting it and the tangency point on the efficient frontier was also "feasible" in the sense that portfolios consisting of some of the riskless asset and some of the market portfolio could be developed. He called that line the CML, and he used indifference curves to show how investors with different degrees of risk aversion would choose portfolios with different mixes of stocks and the riskless asset. Investors who are not at all averse to risk could borrow and buy stocks on margin, and thus move out the CML beyond the tangency point.

Two rational investors could hold portfolios at different points on the CML. An extremely risk averse investor could hold only riskless assets, while someone who is not at all sensitive to risk but who wants to maximize expected returns could move out the CML by buying stock on margin.

Note, though, that all rational investors would have a portfolio that is on the CML. So a highly risk averse investor would not lend up on low-risk stock, nor would a "risk taker" load up on highly risky stocks. Both would invest in the market portfolio and then increase or decrease
3-1  a. A portfolio is made up of a group of individual assets held in combination. An asset that would be relatively risky if held in isolation may have little, or even no risk if held in a well-diversified portfolio.

b. The feasible, or attainable, set represents all portfolios that can be constructed from a given set of stocks. This set is only efficient for part of its combinations.

c. An efficient portfolio is that portfolio which provides the highest expected return for any degree of risk. Alternatively, the efficient portfolio is that which provides the lowest degree of risk for any expected return.

d. The efficient frontier is the set of efficient portfolios out of the full set of potential portfolios. On a graph, the efficient frontier constitutes the boundary line of the set of potential portfolios.

e. An indifference curve is the risk/return trade-off function for a particular investor and reflects that investor’s attitude toward risk. The indifference curve specifies an investor’s required rate of return for a given level of risk. The greater the slope of the indifference curve, the greater is the investor’s risk aversion.

f. The optimal portfolio for an investor is the point at which the efficient set of portfolios—the efficient frontier—is just tangent to the investor’s indifference curve. This point marks the highest level of satisfaction an investor can attain given the set of potential portfolios.

g. The Capital Asset Pricing Model (CAPM) is a general equilibrium market model developed to analyze the relationship between risk and required rates of return on assets when they are held in well-diversified portfolios. The SML is part of the CAPM.

h. The Capital Market Line (CML) specifies the efficient set of portfolios an investor can attain by combining a risk-free asset and the risky market portfolio M. The CML states that the expected return on any efficient portfolio is equal to the riskless rate plus a risk premium, and thus describes a linear relationship between expected return and risk.
i. The characteristic line for a particular stock is obtained by regressing the historical returns on that stock against the historical returns on the general stock market. The slope of the characteristic line is the stock’s beta, which measures the amount by which the stock’s expected return increases for a given increase in the expected return on the market.

j. The beta coefficient (β) is a measure of a stock’s market risk. It measures the stock’s volatility relative to an average stock, which has a beta of 1.0.

k. Arbitrage Pricing Theory (APT) is an approach to measuring the equilibrium risk-return relationship for a given stock as a function of multiple factors, rather than the single factor (the market return) used by the CAPM. The APT is based on complex mathematical and statistical theory, but can account for several factors (such as GNP and the level of inflation) in determining the required return for a particular stock.

l. The Fama-French 3-factor model has one factor for the excess market return (the market return minus the risk-free rate), a second factor for size (defined as the return on a portfolio of small firms minus the return on a portfolio of big firms), and a third factor for the book-to-market effect (defined as the return on a portfolio of firms with a high book-to-market ratio minus the return on a portfolio of firms with a low book-to-market ratio).

m. Most people don’t behave rationally in all aspects of their personal lives, and behavioral finance assumes that investors have the same types of psychological behaviors in their financial lives as in their personal lives.

3-2 Security A is less risky if held in a diversified portfolio because of its lower beta and negative correlation with other stocks. In a single-asset portfolio, Security A would be more risky because σₐ > σₜ and CVₐ > CVₜ.
3-1 a. A plot of the approximate regression line is shown in the following figure:

The equation of the regression line is

$$\bar{Y} = a + b \bar{X}.$$  

The stock's approximate beta coefficient is given by the slope of the regression line:

$$b = \text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{23 - (-14)}{37.2 - (-25.5)} = \frac{37}{63.7} = 0.6.$$  

The intercept, \(a\), seems to be about 3.5. Using a calculator with a least squares regression routine, we find the exact equation to be

$$\bar{Y} = 3.7 + 0.56 \bar{X},$$  

with \(r = 0.96\).
b. The arithmetic average return for Stock X is calculated as follows:

\[
\overline{r}_{mx} = \frac{(-14.0 + 23.0 + \ldots + 18.2)}{7} = 10.6\%.
\]

The arithmetic average rate of return on the market portfolio, determined similarly, is 12.1%.

For Stock X, the estimated standard deviation is 13.1 percent:

\[
\sigma_x = \sqrt{\frac{(-14.0 - 10.6)^2 + (23.0 - 10.6)^2 + \ldots + (18.2 - 10.6)^2}{7 - 1}} = 13.1\%
\]

The standard deviation of returns for the market portfolio is similarly determined to be 22.6 percent. The results are summarized below:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Average return, ( \overline{r}_{mx} )</th>
<th>Standard deviation, ( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock X</td>
<td>10.6%</td>
<td>13.1%</td>
</tr>
<tr>
<td>Market Portfolio</td>
<td>12.1%</td>
<td>22.6%</td>
</tr>
</tbody>
</table>

Several points should be noted: (1) \( \sigma_x \) over this particular period is higher than the historic average \( \sigma_m \) of about 15 percent, indicating that the stock market was relatively volatile during this period; (2) Stock X, with \( \sigma_x = 13.1\% \), has much less total risk than an average stock, with \( \sigma_m = 22.6\% \); and (3) this example demonstrates that it is possible for a very low-risk single stock to have less risk than a portfolio of average stocks, since \( \sigma_x < \sigma_m \).

c. Since Stock X is in equilibrium and plots on the Security Market Line (SML), and given the further assumption that \( \hat{\beta}_x = \hat{\beta}_m \) and \( \hat{\beta}_m = \hat{k}_m \)--and this assumption often does not hold--then this equation must hold:

\[
\overline{r}_x = k_{fr} + (\overline{\beta}_m - k_{fr})\beta_m.
\]

This equation can be solved for the risk-free rate, \( k_{fr} \), which is the only unknown:

\[
10.6 = k_{fr} + (12.1 - k_{fr})0.56
\]

\[
10.6 = k_{fr} + 6.8 - 0.56k_{fr}
\]

\[
0.44k_{fr} = 10.6 - 6.8
\]

\[
k_{fr} = \frac{3.8}{0.44} = 8.6\%.
\]
d. The SML is plotted below. Data on the risk-free security ($k_{fr} = 0$, $k_{mr} = 8.6\%$) and Security X ($b_x = 0.56$, $k_x = 10.6\%$) provide the two points through which the SML can be drawn. $k_m$ provides a third point.

![SML Diagram]

$\text{k} = 12.1\%$

$\text{k} = 8.6\%$

$\text{Beta}$

$\text{Beta}$

2.0

1.0

k(%)

20

10

10

e. In theory, you would be indifferent between the two stocks. Since they have the same beta, their relevant risks are identical, and in equilibrium they should provide the same returns. The two stocks would be represented by a single point on the SML. Stock Y, with the higher standard deviation, has more diversifiable risk, but this risk will be eliminated in a well-diversified portfolio, so the market will compensate the investor only for bearing market or relevant risk. In practice, it is possible that Stock Y would have a slightly higher required return, but this premium for diversifiable risk would be small.
a. The graph is shown above. b will depend on students' freehand line. Using a calculator, we find \( b = 0.62 \).

b. Because \( b = 0.62 \), Stock Y is about 62 percent as volatile as the market; thus, its relative risk is about 62 percent of that of an average firm.

c. 1. Total risk \( \sigma_f \) would be greater because the second term of the firm's risk equation, \( \sigma_f = \beta \sigma_m + \sigma^*_f \), would be greater.

2. CAPM assumes that company-specific risk will be eliminated in a portfolio, so the risk premium under the CAPM would not be affected.

d. 1. The stock's variance would not change, but the risk of the stock to an investor holding a diversified portfolio would be greatly reduced.

2. It would now have a negative correlation with \( k_e \).

3. Because of a relative scarcity of such stocks and the beneficial net effect on portfolios that include it, its "risk premium" is likely to be very low or even negative. Theoretically, it should be negative.

Answers and Solutions: 3 - 10
e. The following figure shows a possible set of probability distributions. We can reasonably surmise that the 100-stock portfolio comprised of $b = 0.62$ stocks as described in Condition 2 will be less risky than the “market.” Hence, the distribution for Condition 2 will be more peaked than that of Condition 3. This statement can also be made on the basis of an analytical approach as shown by the material following the graph.

\[ \tilde{k}_i = \tilde{k}_{100} = \tilde{k}_n = 9.8\% \]
\[ \sigma_i^2 = b_i^2 \sigma_n^2. \]

For Condition 2, with 100 stocks in the portfolio, $\sigma_i^2 \approx 0$, so

\[ \sigma_i^2 = (0.62)^2 \sigma_n^2 \quad \sigma_e = \sqrt{(0.62)^2 \sigma_n^2} = 0.62 \sigma_n. \]

Since $\sigma_p$ is only 62 percent of $\sigma_n$, the probability distribution for Condition 2 is clearly more peaked than that for Condition 3; thus, we can be reasonably confident of the relevant locations of the distributions for Conditions 2 and 3.

With regard to Condition 1, the single-asset portfolio, we can be sure that its probability distribution is less peaked than that for the 100-stock portfolio. Analytically, since $b = 0.62$ both for the single stock portfolio and for the 100-stock portfolio,

\[ \sigma_i^2 = (0.62 \sigma_n)^2 + \sigma_i^2 > (0.62 \sigma_n)^2 + 0 \approx \sigma_i^2. \]

Answers and Solutions: 3 - 11
We can also say on the basis of the available information that \( \sigma_i \) is smaller than \( \sigma_M \). Stock Y's market risk is only 62 percent of the "market," but it does have company-specific risk, while the market portfolio does not. However, we know from the given data that \( \sigma_i = 13.8\% \), while \( \sigma_M = 19.6\% \). Thus, we have drawn the distribution for the single stock portfolio more peaked than that of the market. The relative rates of return are not reasonable. The return for any stock should be

\[
k_i = k_M + (k_M - k_F)b_i \]

Stock Y has \( b = 0.62 \), while the average stock (M) has \( b = 1.0 \); therefore,

\[
k_M = k_M + (k_M - k_F)0.62 < k_M = k_M + |k_M - k_F|1.0.
\]

A disequilibrium exists—Stock Y should be bid up to drive its yield down. More likely, however, the data simply reflect the fact that past returns are not an exact basis for expectations of future returns.

3-3

a. \( k_i = k_M + (k_M - k_F)b_i = k_M + (k_M - k_F)\frac{\bar{x}_{MT}}{\bar{x}_{MT}} \)

b. CML: \( k_i = k_M + (\bar{x}_M - k_M)\frac{\bar{x}_{MT}}{\bar{x}_{MT}} \)

SML: \( k_i = k_M + (\bar{x}_M - k_M)\frac{\bar{x}_{MT}}{\bar{x}_{MT}} \)

With some arranging, the similarities between the CML and SML are obvious. When in this form, both have the same market price of risk, or slope, \((k_M - k_F)/\bar{x}_{MT}\).

The measure of risk in the CML is \( \bar{x}_{MT} \). Since the CML applies only to efficient portfolios, \( \bar{x}_{MT} \) not only represents the portfolio's total risk, but also its market risk. However, the SML applies to all portfolios and individual securities. Thus, the appropriate risk measure is not \( \bar{x}_{MT} \), the total risk, but the market risk, which in this form of the SML is \( r_M\sigma_M \), and is less than for all assets except those which are perfectly positively correlated with the market, and hence have \( r_M = 1.0 \).

3-4

a. Using the CAPM:

\[
k_i = k_M + (k_M - k_F)b_i = 7\% + (1.1)(6.5\%) = 14.5\%
\]

b. Using the 3-factor model:

\[
k_i = k_M + (k_M - k_F)b_i + (K_{PM})\bar{c}_p + (K_{PM})\bar{c}_i = 7\% + (1.1)(6.5\%) + (3\%)(0.7) + (4\%)(-0.3) = 16.45\%
\]

Answers and Solutions: 3 - 12